Math

# Algebra 1

## High School Teacher Edition

Oak Meadow

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Oak Meadow

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Lesson

3

# Part 1: Graphs and Functions

## Exploratory Activity

**(Image credit: Brook Lark)**

Last week, you and a friend went to your favorite cupcake shop with a $10 bill. You found out that each cupcake is $2.25. How many cupcakes were you able to buy?

How did you figure that out?

Complete the following table relating the number of cupcakes to the total cost.

| Number of Cupcakes | Cost in Dollars |
|---|---|
| 1 | 2.25 |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |

| Number of Cupcakes | Cost in Dollars |
|---|---|
| 8 | |
| 9 | |
| 10 | |
| 11 | |
| 12 | |
| $c$ | |

### ASSIGNMENT CHECKLIST

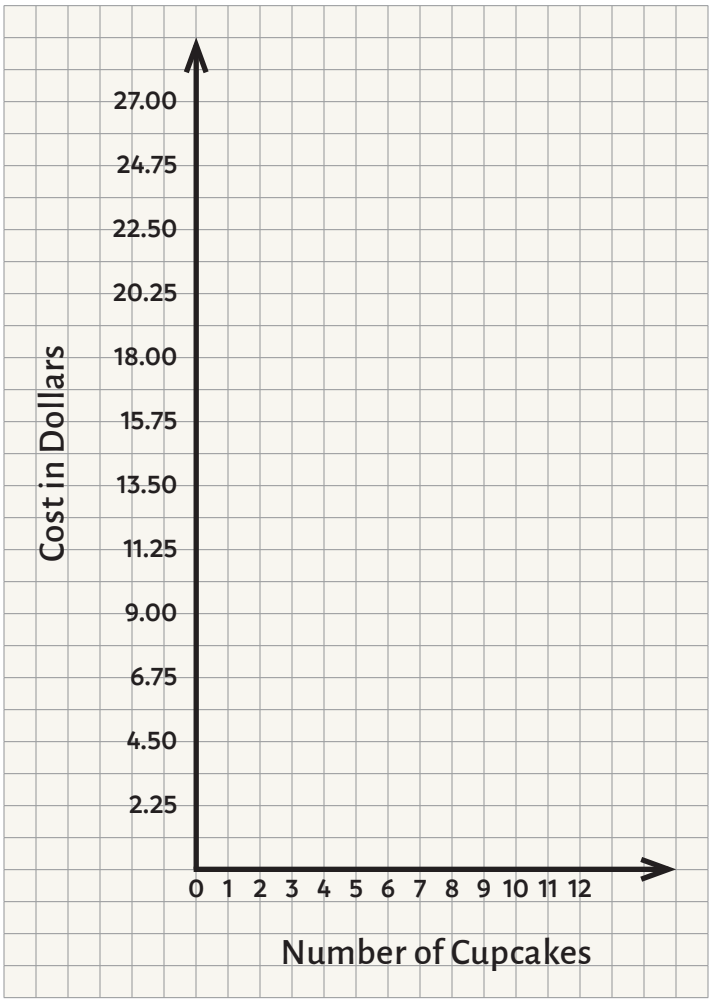
- ☐ Complete the exploratory activity.
- ☐ Complete the assigned problems in Exercise Sets 3.1–3.4.
- ☐ Choose an activity to complete:
  - Activity A: Integrated Review
  - Activity B: Explore with Technology Equations of Lines
  - Activity C: Error Analysis of Slope
  - Activity D: Conduct Your Own Research with Correlation

How much does a dozen (12) cupcakes cost?

What do you notice about the chart?

Could you predict the price for 15 cupcakes? 20 cupcakes? 27 cupcakes?

Plot the points in the table on the graph below. Did you get a straight line?




(As always, you can check your answers in the appendix.)

You may have observed that the points you plotted created a straight line. This is called a **linear relationship**. Notice that as the horizontal axis increases by 1 unit, the vertical axis increases by 2.25 units. This is what we call a **constant rate of change**, which you also might have heard referred to as a **slope**. Chapter 3 focuses on graphs and equations of linear functions, which depend on a constant rate of change.

# Lesson Introduction

**Suggested time: 2 weeks**

Linear functions, like the one in the exploratory activity, are relationships that have a constant rate of change, meaning they increase or decrease by the same amount in each interval. Linear relationships appear all around us. They are a vital tool in real-life applications, and we will see throughout this chapter the many ways they can be used to model real-world scenarios where one variable is dependent on another. Consider the following scenarios: How cold will it get in Antarctica tonight? How much more would I need to make per hour to work fewer hours and net the same wage as I do now? If my friend gets a head start in a race, how fast do I need to run to win? How long will it take an ice cube of a certain size to melt, on average? All these questions are, in essence, situations that involve a constant rate of change. You will learn the skills needed to investigate questions like these in chapter 3.

# Learning Objectives

Use the checklist below to track how your skills are developing over time, and identify skills that need more work as you progress through part 1 and part 2 of this lesson.

| Skills | Notes |
|---|---|
| Graph linear equations and functions | |
| Identify the slope and *y*-intercept of a line from its graph, equation, or points on the line | |
| Find the equation of a line given specific conditions, including a line parallel or perpendicular to another line | |
| Determine if a relation is a function | |
| Find the value of a function for a specific input value | |
| Identify the domain and range of a function from its equation or graph | |
| Interpret and make predictions from a bar graph, a line graph, or an equation | |
| Graph and perform translations of nonlinear functions | |

# Exercise Sets

Read the following sections, and complete the accompanying problem sets. Plan to complete a portion each day. If you have online access to MyMathLab, you can watch the instructional videos as well.

As you complete each set of problems, check your answers using the answer key at the back of the textbook. Correct any problems where you made mistakes. If you need help, let your teacher know.

1. Read section 3.1, "Reading Graphs and the Rectangular Coordinate System" (180), and then complete the following problems in Exercise Set 3.1.
    - 1–33 odd
    - 39–59 EO odd
    - Extension 93–99 all
2. Read section 3.2, "Graphing Linear Equations" (191), and then complete the following problems in Exercise Set 3.2.
    - 1–15 odd
    - 41–53 odd
3. Read section 3.3, "Intercepts" (200), and then complete the following problems in Exercise Set 3.3.
    - 1–29 odd
4. Read section 3.4, "Slope and Rate of Change" (214), and then complete the following problems in Exercise Set 3.4.
    - 1–35 odd
    - 55–65 odd
    - 71 and 73
    - Review and Preview 75 and 77
    - Extension 79 and 81

# Activities

Choose one of the following activities to complete.

- Activity A: Integrated Review
- Activity B: Explore with Technology Equations of Lines
- Activity C: Error Analysis of Slope
- Activity D: Conduct Your Own Research with Correlation

You may choose to answer reflection questions in writing, as an audio recording, or as a video recording. As always, make sure to thoroughly explain your answers.

Activities can be assessed according to the criteria found in the rubric below.

| | Notes |
|---|---|
| **Problem-Solving and Precision**<br>Work is clear, organized, and detailed. Appropriate symbols, labels, units, and terminology are used. | |
| **Reasoning and Explaining**<br>Symbols, words, and diagrams are interpreted with mathematical meaning. Prior knowledge is integrated into reasoning. | |
| **Modeling and Using Tools**<br>Models, tools, and strategies are used to simplify, explain, give structure, and/or communicate a problem-solving strategy and a solution. | |
| **Seeing Structure and Generalizing**<br>Structures and patterns are identified and extended to make generalizations and/or connections to prior learning. | |

### Activity A: Integrated Review

Complete "Integrated Review: Summary on Slope and Graphing Linear Equations" on page 218, problems 1–16 all. Show all your work. Self-check your answers thoroughly, and make corrections to any problems you missed.

### Activity B: Explore with Technology Equations of Lines

1. Open the following link:

   "Partial Variation Warm Up"

2. Fill in the table of values provided, and plot the points of a linear relationship on the graph. Take a screenshot of the resulting graph.
3. Use the table and the graph to find the equation of the linear relationship you created.

4. Repeat the process three more times to create a total of four equations. Take screenshots of each of the graphs you create and include them with your submission.

5. What are the four equations you created? What strategies did you use to build them? Observe the similarities and differences between your graphs and equations. How would you generalize your observations?

   **There are an infiinite number of possible solutions. All equations should have the same *y*-intercept of 4 and a different slope. Students should talk about strategies like noticing patterns within the table of values to create linear graphs and using those graphs to identify the *y*-intercept and slope to create equations.**

   **Example:**

   $y = 2x + 4$

   $y = -3x + 4$

   $y = \frac{1}{4}x + 4$

   $y = -9x + 4$

   **Students should generalize the similarities and differences they observe and notice that all the graphs and equations have the same *y*-intercept and different slopes.**

6. How would the graphs and equations have been different if the given value in the chart was (–1, 4) instead of (0, 4)?

   **Students should observe that if the given point was (–1, 4) instead of (0, 4), the outcome would have been completely different. The *y*-intercept would no longer be a fixed value, and all lines would go through the point (–1, 4) instead, with different *y*-intercepts and different slopes. This would still create a common point in the graph, but the equations would not have any noticeable commonalities in slope-intercept form.**

### Activity C: Error Analysis of Slope

Mallory solved the following problem, but she made two mistakes. Analyze her work to find her mistakes, and then solve the problem correctly.

**Problem:** Find the slope between the points (–1, 2) and (4, 7).

**Mallory's solution:** $m = \frac{4 - -1}{7 - 2}$

$m = \frac{3}{5}$

1. What were Mallory's mistakes?

   **Correct work for Mallory's problem:**

   $$m = \frac{7 - 2}{4 - -1}$$

   $$m = \frac{5}{5}$$

   $$m = 1$$

   **Mallory mistakenly calculated the change in *x* over the change in *y*. The slope formula evaluates the rise over the run, but Mallory calculated the run over the rise, which is incorrect. She also calculated 4 – –1 incorrectly because she forgot that two negative signs next to each other make a positive. She should have calculated 4 + 1 on the top to get 5.**

2. What advice would you give Mallory so that she understands her mistakes and will not make the same mistakes in the future?

   **Advice for Mallory may vary but should include some way to remember that in the slope formula, the change in *y* is on top and the change in *x* is at the bottom.**

Marcus also solved a problem and made two mistakes. Analyze his work to find his mistakes, and then solve the problem correctly.

**Problem:** Find the slope between the points (9, –5) and (–6, 0).

**Marcus's solution:** $m = \frac{0 - -5}{9 - -6}$

$$m = \frac{5}{15}$$

1. What were Marcus's mistakes?

   **Correct work for Marcus's problem:**

   $$m = \frac{0 - -5}{-6 - 9}$$

   $$m = \frac{5}{-15}$$

   $$m = -\frac{1}{3}$$

   **Marcus's first mistake is that even though he put the change in *y* on top and the change in *x* at the bottom, he did not maintain a consistent order between the first point and the second point. On the top, he calculated $y^2 - y^1$; on the bottom, he calculated $x^1 - x^2$. His second mistake was that he did not fully reduce his answer.**

2. What advice would you give Marcus so that he understands his mistakes and will not make the same mistakes in the future?

   **Marcus needs to be consistent and always subtract the first point value from the second point value, or the second point value from the first point value. It does not matter which way he chooses, as long as he does it the same way in the numerator and denominator. He should also always check his answer to make sure it can't be reduced further.**

### Activity D: Conduct Your Own Research with Correlation

Have you ever wondered if something you observe in your own life is positively or negatively correlated? For example, is there a positive correlation between how much you sleep and how fast you can run the next day? Is there a negative correlation between the temperature outside and the amount of hot chocolate you drink? Is there any correlation at all between how many ads you see in a day and how much money you spend online shopping? This activity looks at real-life examples of correlation.

1. Read the following article about real-life examples of positive and negative correlation.

   "6 Examples of Correlation in Real Life"

2. Think of something you wonder about that involves a relationship between two variables you could easily measure. Define your two variables.

   Question/topic: _____

   Variable 1: _____

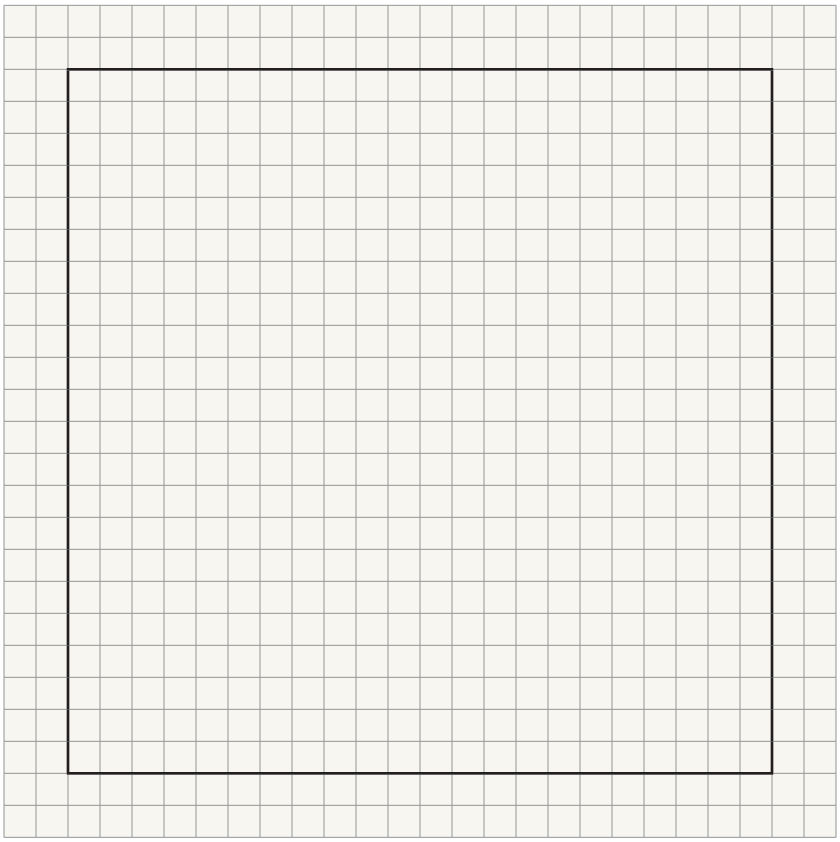
   Variable 2: _____

   Just remember that **correlation does not always mean causation**! The article below demonstrates this.

   "Correlation Does Not Imply Causation: 5 Real-World Examples"

3. Gather data from at least ten people.

4. Plot the data you collected on a clearly labeled coordinate grid. For example, if you want to examine the correlation between sleep and running speed, the first variable would be the number of hours of sleep a person got the night before, and the second variable would be their running speed. You would graph this data by plotting running speed on one axis and hours of sleep on the other to examine the relationship.



5. Answer the following questions.

   **Answers to the questions below will vary. Verify your student's variables are clearly defined and the graph reflects their collected data. The graph should include clearly labeled axes and at least ten data points.**

   a. What do you notice about your data?

      **Answers will vary based on the data collected. Students should correctly identify the type and strength of the correlation. Students should talk about the correlation (or lack thereof) between the two variables they examined and what conclusions they can draw.**

   b. How did you collect the data you used for your experiment?

      **Students should discuss how they collected data, such as interviewing people they know, interviewing strangers, sending out a survey, etc.**

   c. What alterations would you make if you were to do this experiment again or a similar experiment in the future?

      **They should offer suggestions to improve the experiment, such as collecting a larger sample size of data or narrowing down their variables to examine a more specific correlation.**

   d. What additional correlations do you wonder about?

      **Students should offer additional correlations they wonder about and would like to explore.**

### SHARE YOUR WORK

When you have completed this portion of the lesson, please share the following work with your teacher.

- Exercise Sets 3.1–3.4 (showing handwritten computations and corrections)
- Choice of activity (labeled with the title of the activity):
    - Activity A: Integrated Review
    - Activity B: Explore with Technology Equations of Lines
    - Activity C: Error Analysis of Slope
    - Activity D: Conduct Your Own Research with Correlation

Make sure everything is labeled and you've included all your handwritten computations. If you have any questions about the work or how to share it, contact your teacher.

Lesson

3

# Part 2: Graphs and Functions

## Lesson Introduction

**Suggested time: 2 weeks**

Lesson 3 continues with part 2. Refer to part 1 for learning objectives.

In section 3.5, you are going to learn how to write an equation of a line with a given slope through a given point. There are different ways to do this, but the textbook focuses on a single method. Please watch the following video to learn an additional method that you might prefer.

"Algebra 1 Ch 3 Equations of Lines"

Feel free to use either method throughout the rest of the chapter.

## Exercise Sets

Read the following sections, and complete the accompanying problem sets. Plan to complete a portion each day. If you have online access to MyMathLab, you can watch the instructional videos as well.

As you complete each set of problems, check your answers using the answer key at the back of the textbook. Correct any problems where you made mistakes. If you need help, let your teacher know.

1. Read section 3.5, "Equations of Lines" (225), and then complete the following problems in Exercise Set 3.5.
    - ☐ 1–49 EO odd
    - ☐ 69 and 71
    - ☐ 83–86 all
    - ☐ Extension 87–91 odd

### ASSIGNMENT CHECKLIST

- ☐ Complete the assigned problems in Exercise Sets 3.5–3.8.
- ☐ Complete the chapter 3 test.
- ☐ Complete the assessment test (if provided).
- ☐ Choose an activity to complete:
    - Activity A: Investigate Piecewise Functions
    - Activity B: Fill in the Blank—Slopes and Intercepts
    - Activity C: Writing Prompt for Domain and Range
    - Activity D: Get Creative with Lines and Digital Art
    - Activity E: Citizen Math—Linear Equations in Society

2. Read section 3.6, "Functions" (235), and then complete the following problems in Exercise Set 3.6.
    - 1–31 odd
    - 33–65 EO odd
3. Read section 3.7, "Graphing Linear Functions" (242), and then complete the following problems in Exercise Set 3.7.
    - 1–25 odd
    - 39 and 41
    - Review and Preview 45–49 odd
    - Extension 53
4. Read section 3.8, "Graphing Piecewise-Defined Functions and Shifting and Reflecting Graphs of Functions" (252), and then complete the following problems in Exercise Set 3.8.
    - 9–15 odd
    - 17–45 EO odd
    - Review and Preview 49–52 all
    - Extension 55
5. Optional: If you would like more practice, you have the option of completing the following, doing as many problems as needed.
    - Chapter 3 Review and Vocabulary Check (254)
    - Chapter 3 Standardized Test Practice (261)

# Chapter Test

1. In your textbook, complete the chapter 3 test on page 259. After completing the test, you or a supervising adult will grade it and mark the score at the top (for instance, 18/20). Then, review any mistakes and make necessary corrections.
2. For enrolled students: Complete the chapter 3 assessment test (if one has been provided).

# Activities

Choose one of the following activities to complete.

- Activity A: Investigate Piecewise Functions
- Activity B: Fill in the Blank—Slopes and Intercepts
- Activity C: Writing Prompt for Domain and Range

- Activity D: Get Creative with Lines and Digital Art
- Activity E: Citizen Math—Linear Equations in Society

You may choose to answer reflection questions in writing, as an audio recording, or as a video recording. As always, make sure to thoroughly explain your answers.

Activities can be assessed according to the criteria found in the rubric below.

| | **Notes** |
|---|---|
| **Problem-Solving and Precision**<br>Work is clear, organized, and detailed. Appropriate symbols, labels, units, and terminology are used. | |
| **Reasoning and Explaining**<br>Symbols, words, and diagrams are interpreted with mathematical meaning. Prior knowledge is integrated into reasoning. | |
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| **Seeing Structure and Generalizing**<br>Structures and patterns are identified and extended to make generalizations and/or connections to prior learning. | |

### Activity A: Investigate Piecewise Functions

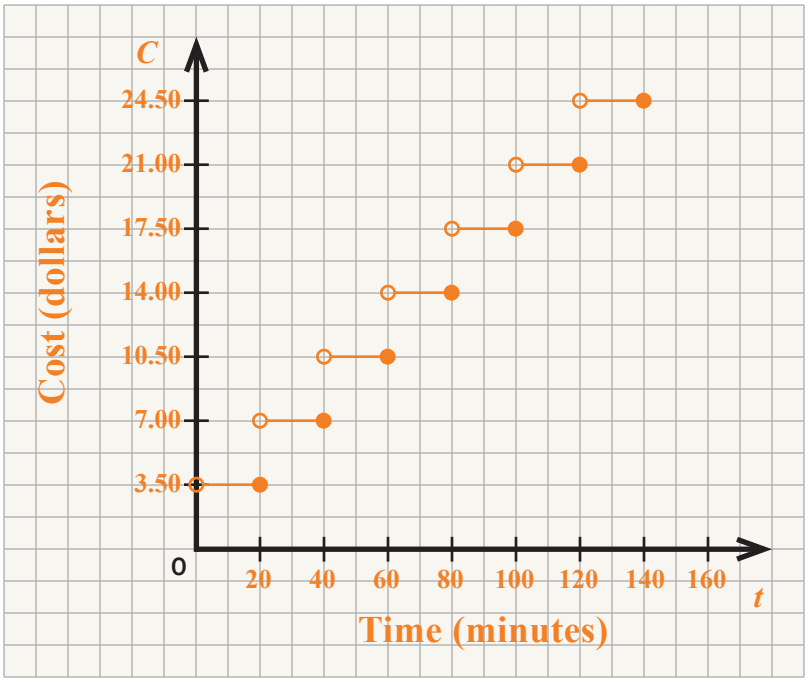
Function *C* represents the cost of a rideshare through a rideshare company for a trip measured in *t* minutes. The following piecewise function defines the cost as a function of time.

$$C(t) = \begin{cases} 3.50, & 0 < t \leq 20 \\ 7.00, & 20 < t \leq 40 \\ 10.50, & 40 < t \leq 60 \\ 14.00, & 60 < t \leq 80 \\ 17.50, & 80 < t \leq 100 \\ 21.00, & 100 < t \leq 120 \\ 24.50, & 120 < t \leq 140 \end{cases}$$

1. Complete the following table relating time and cost.

| *t* (minutes) | *C* (dollars) |
|---|---|
| 4 | 3.50 |
| 23 | 7.00 |
| 35 | 7.00 |
| 56 | 10.50 |
| 60 | 10.50 |
| 81 | 17.50 |
| 99 | 17.50 |
| 117 | 21.00 |
| 134 | 24.50 |

2. Sketch the graph of $C$ for all values of $t$ where $0 < t \leq 140$. Clearly label both axes.




3. Describe the pricing model of this rideshare company.

**The pricing model of this rideshare company is to charge riders $3.50 for every 20 minutes they spend in the vehicle. Their pricing is based on time spent in the car, not distance traveled. The pricing also jumps by a flat rate of $3.50 every 20 minutes without increasing within the 20-minute intervals.**

4. Define the domain and range of this function, and explain what they each represent.

   **The domain of this function is $0 < x \leq 140$. The domain represents the restrictions on time in this situation. Time starts at 0 and ends at 140 minutes, based on the given restrictions, which means you can ride in the car for up to 140 minutes.**

   **The range of this function is the set of $y$-values such that $y = \{3.5, 7, 10.5, 14, 17.5, 21, 24.5\}$. Range represents the restrictions on cost in this situation. Because the function is not continuous and increases by $3.50 every 20 minutes, $y$-values (cost) can only exist at those specific values.**

### Activity B: Fill in the Blank—Slopes and Intercepts

1. Use the integers –9 through 9, no more than once each, to fill in the boxes to create a linear equation that has one positive and one negative intercept with integer values (a positive $x$-intercept and a negative $y$-intercept or a positive $y$-intercept and a negative $x$-intercept).

   $$\square x + \square y = \square$$

   **There are many possible solutions. No digit should be used twice. The number in the last box should be evenly divisible by both numbers in the first two boxes, and one of the numbers on the left-hand side should have a negative value so that one intercept is positive and one intercept is negative. If only the right-hand side contains a negative value, then both intercepts would be negative, which is incorrect. If the student placed a negative value in one of the left-hand boxes in addition to the right-hand side, then the answer would be correct.**

   **Examples:**

   **$-2x + 4y = 8$**

   **$3x + -1y = 9$**

   **$-3x + 2y = -6$**

2. Now, use the integers –9 through 9, no more than once each, to create an equation of a line that is perpendicular to the equation you created in step 1. (You can repeat a digit from your first equation.)

   $$\square x + \square y = \square$$

   **The solution should be based on the equation students created in step 1. No digit should be repeated, and the slope should be the opposite reciprocal of the slope in the first equation.**

   **Examples based on the initial equations given:**

   **$-2x + 4y = 8$ is perpendicular to $6x + 3y = 9$**

   **$3x + -1y = 9$ is perpendicular to $-1x + 3y = 6$**

   **$-3x + 2y = -6$ is perpendicular to $2x + 3y = 9$**

3. Then, answer the following questions.

   a. What are your intercepts, and how did you ensure one was positive and one was negative for the first equation?

   **Students should state their intercepts and describe how they ensured one was positive and one was negative, such as something along the lines of the description given above.**

   b. What was the slope of your first line? What was the slope of your second line?

   **Students should state the slopes of their two lines.**

   c. What strategies did you use to find your solution?

   **Possible strategies include guess and check, and playing around with signs and divisibility. They likely converted their first equation to slope-intercept form to identify the slope. It's possible they may have found the perpendicular slope and worked backward from slope-intercept form to create an equation in standard form. They also might have experimented with different $y$-intercept values in their second equation. Students should mention any strategies they used to ensure each digit was used only once.**

### Activity C: Writing Prompt for Domain and Range

For this activity, you may graph by hand or use a graphing technology like Desmos.com. You will submit your equations and graphs for each part.

**Students should submit their equations and their graphs for each part of the problem.**

1. Create two lines that have the same domain and the same range. State the domain and range.

   **Any two non-vertical and non-horizontal lines will have the same domain (all real numbers) and the same range (all real numbers).**

   **Sample solution:**

   **The equations $y = 2x + 3$ and $y = -\frac{1}{4}x - 1$ both have a domain and range of all real numbers.**

2. Create two lines that have the same domain but different ranges. State the domain and range.

   **Any two horizontal lines will have the same domain (all real numbers) but different ranges ($y = \#$).**

   **Sample solution:**

   **The equations $y = 4$ and $y = -2$ both have a domain of all real numbers but a different range ($y = 4$ and $y = -2$, respectively).**

3. Create two lines that have the same range but different domains. State the domain and range.

   **Any two vertical lines will have the same range (all real numbers) but different domains ($x = \#$).**

**Sample solution:**

**The equations $x = 3$ and $x = -10$ both have a range of all real numbers but a different domain ($x = 3$ and $x = -10$, respectively).**

4. Complete the following generalizations.

   a. Any two lines will have the same domain and range if . . .

      **Any two lines will have the same domain and range if they are non-vertical and non-horizontal.**

   b. Any two lines will have the same domain but different ranges if . . .

      **Any two lines will have the same domain but different ranges if they are horizontal.**

   c. Any two lines will have the same range but different domains if . . .

      **Any two lines will have the same range but different domains if they are vertical.**

5. Can two lines have a different domain and a different range? Explain.

   **Students should conjecture that two lines could have a different domain and a different range if one was horizontal (domain = all real numbers, range = #) and one was vertical (domain = #, range = all real numbers). As long as they were going through different *x*- and *y*-values, the range # and domain # would also be different.**

6. What was your favorite part of this activity?

   **Students should describe something they enjoyed about this activity.**

### Activity D: Get Creative with Lines and Digital Art

In this activity, you will create a digital art piece using your knowledge of linear equations, piecewise functions, and domain and range.

Your picture could be as simple as the examples below or as intricate as you would like.

"Algebra 1 Chapter 3 Part 2 Option 4 Example"

1. Begin by opening the following link:

   "Graphing Calculator"

   Your picture must contain the following:

   - At least 8 lines total
   - At least 1 pair of parallel lines
   - At least 1 pair of perpendicular lines
   - At least 1 line with a positive slope

- At least 1 line with a negative slope
- At least 1 line with an undefined slope
- At least 1 line with a zero slope

Use the example above to help you figure out how to restrict the domain or range of your line to cut it off where you would like. Simply type in the equation of the line and use brackets—{ }—and inequality symbols—< and >—to restrict the domain or range. For more tips, check out the resource below:

"Getting Started: Inequalities and Restrictions"

2. Save your graph and share the link with your teacher so they can examine your picture and all your equations. (You may also create a screenshot of your work as long as you include the whole picture and all your equations.)

   **See the link at the beginning of the activity for an example of the type of design your student could create. Students must meet the minimum line requirements listed, and they should have the properly restricted domain and range for each of their equations.**

3. What is your favorite part of your creation?

   **Students should describe their favorite aspect of their creation.**

### Activity E: Citizen Math—Linear Equations in Society

Complete the following Citizen Math activity to explore some of the factors related to homelessness by focusing on data from one of the largest cities in the United States. You can choose to investigate Los Angeles, Seattle, or New York. You will apply your math skills to real-world data and consider how this data can be used to find solutions.

Often, in real life, data does not create a perfect line. When we look at real data and try to describe it with a linear equation, we sometimes use a line of best fit. This is just what it sounds like: a linear relationship that best represents the data. It is the best approximation, not an exact description, of the data. You will work with lines of best fit in this activity. Feel free to reach out to your teacher if you have any questions about this.

1. First, watch this short video to get started:

   "Cities Compete for Amazon HQ2"

   If you were the mayor of a town or city, would you want Amazon to build its new headquarters in your area? What might be the benefits and challenges of having a large international company like Amazon based in your town?

2. Choose which city to focus on—Los Angeles, Seattle, or New York—and view the relevant link below. You can print out the worksheet and complete the work directly on the page or do your work on a separate sheet of paper.

   "Seeking Shelter: Los Angeles"

   "Seeking Shelter: Seattle"

   "Seeking Shelter: New York"

3. Complete questions 1 and 2, then watch the following video before completing questions 3 and 4.

   "Homelessness in America & Rising Rent"

   **You can find the answer key at oakmeadow.com/answer-keys.**

4. Optional extension: There are many factors that impact homelessness in communities. Social services and affordable housing are two major influencers. Even within a city, certain neighborhoods are disproportionately affected by rising costs. When large company headquarters arrive in communities, they primarily offer lower-level positions at low wages along with a limited number of high-paying executive jobs, which perpetuates class divisions and impacts the economy of the community. Oftentimes, people can no longer afford to live in their communities and are displaced by those with higher incomes who can afford to live there, which is a process called *gentrification*. Since 2019, the cost of rent and housing has skyrocketed across the United States, and it has had a dramatic impact on the unhoused population in this country.

   Can you find current data for the city that you focused on or the city you live in (or near)?

   a. Research the current median annual rent, median annual income, and the homelessness rate.

   b. Create a graph that compares the median annual rent with the median annual income.

   c. Create a graph that compares the rate of homelessness in the area with the median annual rent.

   **Answers to the extension activity will vary. Verify your student's work. If the graphs appear to show unusual results, ask to see the student's sources of data and discuss how they arrived at their results.**

### SHARE YOUR WORK

When you have completed this portion of the lesson, please share the following work with your teacher.

- Exercise Sets 3.5–3.8 (showing handwritten computations and corrections)
- Chapter 3 test

- Chapter 3 assessment test (if one has been provided).
- Choice of activity (labeled with the title of the activity):
    - Activity A: Investigate Piecewise Functions
    - Activity B: Fill in the Blank—Slopes and Intercepts
    - Activity C: Writing Prompt for Domain and Range
    - Activity D: Get Creative with Lines and Digital Art
    - Activity E: Citizen Math—Linear Equations in Society

Make sure everything is labeled and you've included all your handwritten computations. If you have any questions about the work or how to share it, contact your teacher.

Lesson

4

# Part 1: Solving Inequalities and Absolute Value Equations

## Exploratory Activity

Is the following statement true or false?

$$10 < 20$$

Justify your answer.

If we divided both sides of the inequality by 5, what would the result be?

Is this statement still true?

Let's start with our original inequality again, 10 < 20. If we divided both sides by –5 this time, what would the result be?

Is this statement still true?

You may have observed that dividing both sides by –5 resulted in a false inequality because –2 is not less than –4. What do you think we could do to make this inequality true?

If we flip the inequality sign, we can restore the truth value to the statement because we would get –2 > –4, which is accurate. This is how we arrive at the following rule when solving inequalities:

> When we multiply or divide both sides of an inequality by a negative value, we must flip the inequality sign.

Note that –2 > –4 is an equivalent statement to –4 < –2. In the second statement, the sign did not flip, but the numbers did. Just keep this in mind as you explore this chapter. There is often more than one way to write an inequality statement correctly!

### ASSIGNMENT CHECKLIST

- ☐ Complete the exploratory activity.
- ☐ Complete the assigned problems in Exercise Sets 4.1 and 4.2.
- ☐ Choose an activity to complete:
  - Activity A: Integrated Review
  - Activity B: Fill in the Blank—Linear Inequalities
  - Activity C: Writing Prompt for Compound Inequalities

# Lesson Introduction

**Suggested time: 1.5 weeks**

Which is more? An equation won't answer this question, but an inequality will. In this lesson, we will begin comparing quantities that are not equal. Inequalities are important because they allow for a range of values that could be solutions to a problem. They introduce a limit to the value of a variable rather than presenting us with a single exact value. If you want to figure out how many hours you have to work to make at least $200, the greatest number of an item you can buy with a $20 bill, or how far you can drive on three gallons of gas, you likely need to solve an inequality to find a set of solutions rather than a single answer.

# Learning Objectives

Use the checklist below to track how your skills are developing over time, and identify skills that need more work as you progress through part 1 and part 2 of this lesson.

| Skills | Notes |
|---|---|
| Solve and graph linear inequalities in one variable on a number line | |
| Find the intersection and union of two sets in the form of compound inequalities | |
| Solve absolute value equations and inequalities | |
| Solve and graph linear inequalities in two variables on a coordinate plane | |

# Exercise Sets

Read the following sections, and complete the accompanying problem sets. Plan to complete a portion each day. If you have online access to MyMathLab, you can watch the instructional videos as well.

As you complete each set of problems, check your answers using the answer key at the back of the textbook. Correct any problems where you made mistakes. If you need help, let your teacher know.

1. Read section 4.1, "Linear Inequalities and Problem Solving" (272), and then complete the following problems in Exercise Set 4.1.
    - ☐ 1–27 odd
    - ☐ 51 and 53
    - ☐ Review and Preview 61, 63, and 65

2. Read section 4.2, "Compound Inequalities" (278), and then complete the following problems in Exercise Set 4.2.
    - ☐ 1–41 EO odd
    - ☐ Review and Preview 73–79 odd

# Activities

Choose one of the following activities to complete.

- Activity A: Integrated Review
- Activity B: Fill in the Blank—Linear Inequalities
- Activity C: Writing Prompt for Compound Inequalities

You may choose to answer reflection questions in writing, as an audio recording, or as a video recording. As always, make sure to thoroughly explain your answers.

Activities can be assessed according to the criteria found in the rubric below.

| | **Notes** |
|---|---|
| **Problem-Solving and Precision**<br>Work is clear, organized, and detailed. Appropriate symbols, labels, units, and terminology are used. | |
| **Reasoning and Explaining**<br>Symbols, words, and diagrams are interpreted with mathematical meaning. Prior knowledge is integrated into reasoning. | |
| **Modeling and Using Tools**<br>Models, tools, and strategies are used to simplify, explain, give structure, and/or communicate a problem-solving strategy and a solution. | |
| **Seeing Structure and Generalizing**<br>Structures and patterns are identified and extended to make generalizations and/or connections to prior learning. | |

### Activity A: Integrated Review

Complete "Integrated Review: Linear and Compound Inequalities" on page 280, problems 1–12 all. Show all your work. Self-check your answers thoroughly, and make corrections to any problems you missed.

### Activity B: Fill in the Blank—Linear Inequalities

Use the digits 0 through 9 no more than once each to create an inequality that is equal to $x > -3$.

$$\square x - \square < \square x + \square$$

What strategies did you use to find your solution?

**There are many possible solutions. Verify your student's work. Some possible answers include:**

**$3x - 2 < 5x + 4$**

**$4x - 1 < 6x + 5$**

**$2x - 0 < 5x + 9$**

**$5x - 2 < 7x + 4$**

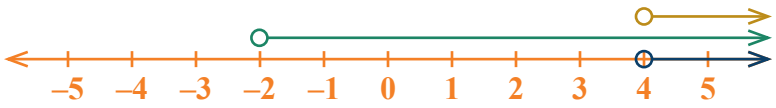
**Students should describe strategies they used to solve this problem. Perhaps they noticed that they needed to flip the inequality sign by dividing by a negative number, so they ensured the value connected to the $x$ on the left-hand side was less than the value connected to the $x$ on the right-hand side. They might have strategized to create an $x$ value like $-2x$ or $-3x$ on the left-hand side and then created a 6 or a 9 with the other two values on the right-hand side to result in a quotient of –3.**

### Activity C: Writing Prompt for Compound Inequalities

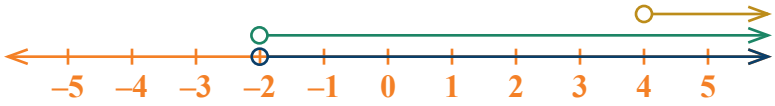
Think about the definitions of *union* and *intersection* in relation to "and" and "or" when you consider these problems.

1. If the solution to a compound inequality with an "and" statement was $x > -2$ and $x > 4$, what is the answer? Graph the given solution on a number line, and thoroughly explain what the answer is and why.

   **Because this is an "and" statement, we are looking for the intersection, or overlap, of the two solutions. The solutions begin their overlap after 4; therefore, the correct solution is $x > 4$ only. Prior to 4, only one of the inequalities is satisfied, which does not create a union.**



2. If the solution to a compound inequality with an "or" statement was $x > -2$ and $x > 4$, what is the answer? Graph the given solution on a number line, and thoroughly explain what the answer is and why.




**Because this is an "or" statement, we are looking for the union of the two solutions. This is where one, or the other, or both inequalities are satisfied. Starting at –2 and to the right, one or the other or both equations are satisfied. Therefore, the correct solution is $x > -2$. To include $x > 4$ is redundant and unnecessary.**

3. Can you think of an instance where the answer to an "and" compound inequality would be "no solution"? Give an example that would satisfy this requirement. Graph it and thoroughly explain why there is no solution.


**No Solution**

**Students should create an example in which there is no overlap between the two solution sets and graph it on a number line. One example is $x < -2$ and $x > 4$. (Verify your student's solutions.) There is no point that is an element of both solutions. Therefore, no union exists, and there is no solution.**

4. Can you think of an instance where the answer to an "or" compound inequality would be "all real numbers"? Give an example that would satisfy this requirement. Graph it and thoroughly explain why the solution is "all real numbers."


**All Real Numbers**

**Students should create an example in which every number on the number line is covered at least once by a solution and graph it on a number line. One example is $x > -2$ or $x < 4$. (Verify your student's solution.) Every point on the number line is covered at least once, which means the intersection of the two solution sets is all real numbers.**

### SHARE YOUR WORK

When you have completed this portion of the lesson, please share the following work with your teacher.

- Exercise Sets 4.1 and 4.2 (showing handwritten computations and corrections)
- Choice of activity (labeled with the title of the activity):
    - Activity A: Integrated Review
    - Activity B: Fill in the Blank—Linear Inequalities
    - Activity C: Writing Prompt for Compound Inequalities

Make sure everything is labeled and you've included all your handwritten computations. If you have any questions about the work or how to share it, contact your teacher.

Lesson

4

# Part 2: Solving Inequalities and Absolute Value Equations

## Lesson Introduction

**Suggested time: 1.5–2 weeks**

Lesson 4 continues with part 2. Refer to part 1 for learning objectives.

## Exercise Sets

Read the following sections, and complete the accompanying problem sets. Plan to complete a portion each day. If you have online access to MyMathLab, you can watch the instructional videos as well.

As you complete each set of problems, check your answers using the answer key at the back of the textbook. Correct any problems where you made mistakes. If you need help, let your teacher know.

1. Read section 4.3, "Absolute Value Equations" (285), and then complete the following problems in Exercise Set 4.3.
    - ☐ 1–25 odd
    - ☐ 59–73 odd
    - ☐ Extension 83, 85, and 87
2. Read section 4.4, "Absolute Value Inequalities" (290), and then complete the following problems in Exercise Set 4.4.
    - ☐ 29–61 EO odd
    - ☐ Extension 93, 95, and 97
3. Read section 4.5, "Graphing Linear Inequalities" (296), and then complete the following problems in Exercise Set 4.5.
    - ☐ 1–45 EO odd
    - ☐ 47
    - ☐ Review and Preview 49–57 odd

### ASSIGNMENT CHECKLIST

- ☐ Complete the assigned problems in Exercise Sets 4.3–4.5.
- ☐ Complete the chapter 4 test.
- ☐ Complete the assessment test (if provided).
- ☐ Choose an activity to complete:

    Activity A: Get Creative with Graphs of Inequalities

    Activity B: Which One Doesn't Fit? Linear Inequalities

    Activity C: Real-World Application of Absolute Value Inequalities

    Activity D: Graphing a Budget Equation

4. Optional: If you would like more practice, you have the option of completing the following, doing as many problems as needed.
    - ☐ Chapter 4 Review and Vocabulary Check (298)
    - ☐ Chapter 4 Standardized Test Practice (299)

# Chapter Test

1. In your textbook, complete the chapter 4 test on page 299. After completing the test, you or a supervising adult will grade it and mark the score at the top (for instance, 18/20). Then, review any mistakes and make necessary corrections.
2. For enrolled students: Complete the chapter 4 assessment test (if one has been provided).

# Activities

Choose one of the following activities to complete.

- Activity A: Get Creative with Graphs of Inequalities
- Activity B: Which One Doesn't Fit? Linear Inequalities
- Activity C: Real-World Application of Absolute Value Inequalities
- Activity D: Graphing a Budget Equation

You may choose to answer reflection questions in writing, as an audio recording, or as a video recording. As always, make sure to thoroughly explain your answers.

Activities can be assessed according to the criteria found in the rubric below.

| | **Notes** |
|---|---|
| **Problem-Solving and Precision**<br>Work is clear, organized, and detailed. Appropriate symbols, labels, units, and terminology are used. | |
| **Reasoning and Explaining**<br>Symbols, words, and diagrams are interpreted with mathematical meaning. Prior knowledge is integrated into reasoning. | |
| **Modeling and Using Tools**<br>Models, tools, and strategies are used to simplify, explain, give structure, and/or communicate a problem-solving strategy and a solution. | |

| | Notes |
|---|---|
| **Seeing Structure and Generalizing**<br>Structures and patterns are identified and extended to make generalizations and/or connections to prior learning. | |

## Activity A: Get Creative with Graphs of Inequalities

1. Begin by opening the following graphing calculator:

   "Graphing Calculator"

2. Graph a pentagon (5 sides), hexagon (6 sides), or octagon (8 sides) using inequalities. The inside of your shape should be shaded from all the overlapping regions of your inequalities. Note these shapes do not need to be regular, meaning all sides do not need to be the same length, but your shapes should have symmetry. You can learn how to restrict your shading for both the domain and range in the video tutorial below:

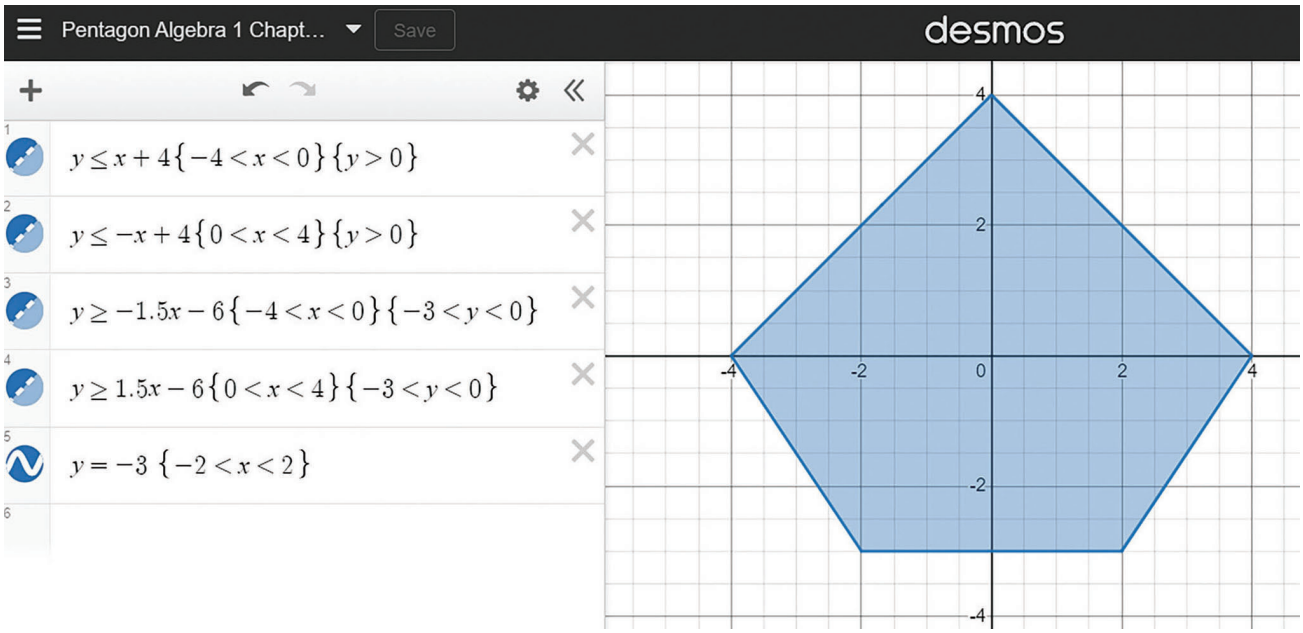
   "Restrictions in Desmos 5 Shade a Restricted Equation, Line"

   Here is another useful resource:

   "Getting Started: Inequalities and Restrictions"

   Tip: You can change the color of each line/shading by clicking and holding down on the color squiggle next to the equation and selecting a new color.

   **There are many possible solutions. Shapes should have some kind of symmetry, but they do not necessarily need to be regular. Shading should be restricted to the interior of the shape, and inequalities and equations should all be correct. One possible solution for a pentagon is shown below. Note symmetry, inequalities, shading, and restrictions of both domain and range.**




## Activity B: Which One Doesn't Fit? Linear Inequalities

| | |
|---|---|
| A<br>$x + 3 \ge 6$ | |
| B<br>$-2x + 3y < 6$ | |
| C<br>$y - 2 \ge 0$ | |
| D<br>$2x - 3y \le 6$ | |

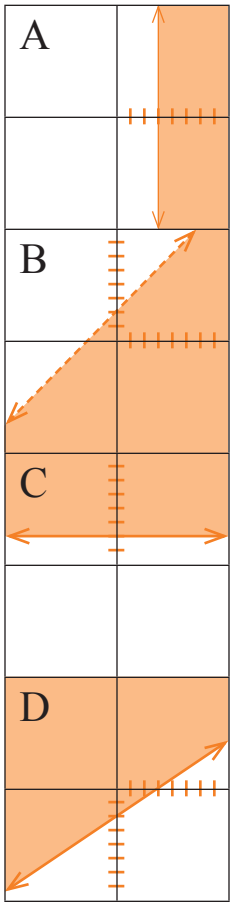
| | |
|---|---|
| A | |
| | |
| B | |
| | |
| C | |
| | |
| D | |
| | |

1. Examine the inequalities in the left-hand column.
2. Sketch a graph of each one in the corresponding box in the right-hand column.
3. Find at least one reason why each inequality/graph could be a correct answer to the question "Which one doesn't fit?"

   Explain your reasoning for each choice and submit them with your graphs.

**There are many possible solutions. Students should begin by correctly graphing each inequality as shown below.**

| | |
|---|---|
| A<br>$x + 3 \geq 6$<br>$x \geq 3$ | A |
| B<br>$-2x + 3y < 6$<br>$y < \frac{2}{3}x + 2$ | B |
| C<br>$y - 2 \geq 0$<br>$y \geq 2$ | C |
| D<br>$2x - 3y \leq 6$<br>$y \geq \frac{2}{3}x - 2$ | D |



**Students should come up with at least one reason why each inequality/graph does not fit with the others. Ideally, these would be reasons specific to inequalities and not just linear equations in general (slopes and *y*-intercepts), although those are valid observations as well. Possible answers include:**

- **A does not fit because it is the only inequality that is a vertical line. It is also the only one with an undefined slope and no *y*-intercept.**
- **B does not fit because it is the only inequality that is non-inclusive and therefore has a dotted line. It is also the only inequality that represents "less than" and has a shaded region below the line.**

- **C does not fit because it is the only inequality that is a horizontal line. It is also the only one with a zero slope and no $x$-intercept.**
- **D does not fit because it is the only inequality that requires the sign to be flipped when simplifying. The solutions are actually greater than the line, not less than. It is also the only one with a negative $y$-intercept.**

### Activity C: Real-World Application of Absolute Value Inequalities

1. Watch the following video:

   "Absolute Value Real-World Applications"

   Use what you learned to respond to the following problems, showing all your work.

2. A car company has pledged to ensure all its cars have an average gas mileage of 32.5 miles per gallon with a variance of no more than 2.5 miles per gallon.

   a. Write an absolute value inequality to model this situation.

   **$|x - 32.5| \leq 2.5$**

   b. What is the range of acceptable miles per gallon for this company's fleet of cars? Express your answer as a compound inequality.

   **$x - 32.5 \leq 2.5$** **$-(x - 32.5) \leq 2.5$**

   **$x \leq 35.00$** **$-x + 32.5 \leq 2.5$**

   **$-x \leq -30.00$**

   **$x \geq 30.00$**

   **$30 \leq x \leq 35$ *miles per gallon***

   c. What change would you make to the inequality to find the range of unacceptable miles per gallon for this company's fleet of cars?

   **Flip the inequality sign. $|x - 32.5| \geq 2.5$**

3. A scientist is working in a climate-controlled laboratory that needs to stay at 65°F with a tolerance of 1.25°F.

   a. Write an absolute value inequality to model this situation.

   **$|x - 65| \leq 1.25$**

b. What is the range of acceptable temperatures for the laboratory? Express your answer as a compound inequality.

$x - 65 \leq 1.25$ $\quad -(x - 65) \leq 1.25$

$x \leq 66.25$ $\quad -x + 65 \leq 1.25$

$-x \leq -63.75$

$x \geq 63.75$

$63.75° \text{ F} \leq x \leq 66.25° \text{ F}$

c. What is the range of unacceptable temperatures for the laboratory?

$|x - 65| \geq 1.25$

$x \leq 63.75° \text{ F}$ *or* $x \geq 66.25° \text{ F}$

4. A company has decided to pay all its employees an annual salary between $55,000 and $80,000. Write an absolute value inequality to model this situation.

$55{,}000 \leq x \leq 80{,}000$

$\textit{average salary} = \frac{55{,}000 + 80{,}000}{2} = \$67{,}000$

$\textit{tolerance} = 80{,}000 - 67{,}500 = \$12{,}500$

$|x - 67{,}500| \leq 12{,}500$

### Activity D: Graphing a Budget Equation

Complete the following activity from Next Gen Personal Finance to explore systems of linear equations and inequalities in real-world finance. This activity covers writing and interpreting linear equations and inequalities, their equations and graphs, budgeting, and comparison shopping.

1. Open the following document:

   "Math: Graphing a Budget Equation"

2. Read the introduction, and watch the EdPuzzle video linked in the upper right corner.
3. Study the example problem in part 1 of the document, and then complete the practice problems in part 2 and the reflection questions in part 3. You can either print this document and complete your work directly on it or use a separate sheet of paper to answer all the questions.

   Remember to show all your work and answer the reflection questions.

   **You can find the answer key at oakmeadow.com/answer-keys.**

### SHARE YOUR WORK

When you have completed this portion of the lesson, please share the following work with your teacher.

- Exercise Sets 4.3–4.5 (showing handwritten computations and corrections)
- Chapter 4 test
- Chapter 4 assessment test (if one has been provided).
- Choice of activity (labeled with the title of the activity):
    - Activity A: Get Creative with Graphs of Inequalities
    - Activity B: Which One Doesn't Fit? Linear Inequalities
    - Activity C: Real-World Application of Absolute Value Inequalities
    - Activity D: Graphing a Budget Equation

Make sure everything is labeled and you've included all your handwritten computations. If you have any questions about the work or how to share it, contact your teacher.