

grade 7

MATH  
COURSEBOOK  
SAMPLE



**oak meadow**

INDEPENDENT LEARNING SINCE 1975

# Grade 7 Math

## Oak Meadow Coursebook

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Grade



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# Grade 7 Dividing Decimals; Factors and Prime Numbers



## Mental Math

As you perform your mental math, if you ever want to check your answer on paper, please do. It's not necessary, though—even if you get the answer wrong, just the act of doing calculations in your head will strengthen your math skills, even if you make a mistake now and then.

**Version 1:** Ask someone to give you a two-digit number. Remember this number, and then reverse the digits to create a second number. Add the two numbers together. For instance, if the original number is 83, you add  $83 + 38 = 121$ . Do this several times. See if you notice a pattern. Then ask for a three-digit number and do the same (for instance,  $168 + 861$ ). Solve several three-digit problems. Feel free to challenge yourself with four-digit numbers, too!

**Version 2:** Ask someone for a two-digit number. Repeat this number aloud. Then in your head, create a new number by using the same digits again to form a four-digit number. Say this number aloud, and then add the same two-digits again to create a six-digit number, and say this number aloud. See if you can go up to a ten-digit number (billions place) or further. For instance, if the original number is 98, then the four-digit number is 9,898 (nine thousand, eight hundred ninety-eight); the six-digit number is 989,898 (nine hundred eighty-nine thousand, eight hundred ninety-eight); and so on.

## Skills Check

Complete the following worksheet to practice some of the skills you have learned.

- Lesson 6 Skills Check

## ASSIGNMENT SUMMARY

- Play mental math games.
- Complete the Skills Check worksheet.
- Read New Skills instruction.
- Complete New Skills Practice.
- Complete Lesson 6 Test and Learning Checklist.



## Lesson 6 New Skills

(continued)

### Dividing Decimals by Whole Numbers

When dividing decimals, the process is the same as dividing whole numbers, except we have to account for the decimal point. The only difference is an adjustment we make to put the decimal point in the correct place.

**Example:**  $42.93 \div 3$

**Step 1:** Rewrite the problem using the division bracket. Remember that the number being divided (which goes inside the bracket) is the dividend; the number doing the dividing (which goes outside the bracket) is the divisor; and the answer is the quotient.

$$3 \overline{)42.93}$$

**Step 2:** Divide as usual, keeping the digits in the correct columns. Ignore the decimal point for now.

**Step 3:** To create the final answer, place the decimal point in the answer directly above where it is in the dividend.

$$\begin{array}{r} 14.31 \\ 3 \overline{)42.93} \\ \underline{3} \phantom{00} \\ 12 \phantom{00} \\ \underline{12} \phantom{00} \\ 09 \phantom{00} \\ \underline{9} \phantom{00} \\ 03 \phantom{00} \\ \underline{3} \phantom{00} \\ 0 \end{array}$$

You use this same process no matter how many decimal places there are in the dividend:

## Dividing with Dividends Less Than 1

When you divide a whole number into a dividend that is less than one, you have to pay attention to where the zeros are, and make sure the zeros are placed correctly in the quotient.

**Example:**  $0.25 \overline{) 5}$

$$5 \overline{) 0.25}$$

**Step 1:** Treat the zero just like any other number. Say to yourself, “How many times does 5 go into 0?” Since 5 doesn’t go into 0, the answer is 0, so you write that in the quotient, then multiply, subtract, and bring down the next digit (the 2) as usual. Repeat the long division process of divide, multiply, subtract, and bring down until you have solved the problem.

**Step 2:** Place the decimal point in the answer so that it directly lines up with the decimal point in the dividend.

$$\begin{array}{r} 0.05 \\ 5 \overline{) 0.25} \\ \underline{0} \\ 02 \\ \underline{0} \\ 25 \\ \underline{25} \\ 0 \end{array}$$

## Dividing Decimals with Remainders

When you have a remainder in a decimal division problem, you continue to add zeros to the end of the dividend until there is no remainder. Adding zeros to the end of the dividend doesn’t change the value of the dividend, it just renames it, which allows us to complete the problem.

**Example:**  $8 \overline{) 7.31}$

**Step 1:** Divide as usual, keeping the digits in the correct columns. When you end up with a remainder (3), add a zero to the

## Lesson 6

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## Lesson 6

*(continued)*

dividend, bring it down, and continue dividing as usual. Continue dividing, adding zeros to the dividend and bringing them down until the answer comes out evenly and there is no remainder left. When you finish dividing, bring the decimal point directly up into the quotient for the final answer.

$$\begin{array}{r}
 0.91375 \\
 \hline
 8 \overline{)7.31000} \\
 \underline{0} \\
 73 \\
 \underline{72} \\
 11 \\
 \underline{8} \\
 30 \\
 \underline{24} \\
 60 \\
 \underline{56} \\
 40 \\
 \underline{40} \\
 0
 \end{array}$$

**Rounding Decimals**

Some decimal problems end up with answers that involve many decimal places. Since an answer in hundredths (two decimal places) is sufficient for most problems, the best solution is to round off the answer to two decimal places. In order to do that, you have to solve the problem through the thousandths place, and if there is still a remainder, follow the basic rules of rounding:

1. **If the digit in the thousandths place is 5 or greater, drop it and increase the digit in the hundredths place by 1**
2. **If the digit in the thousandths place is less than 5, drop it and keep the digit in the hundredths place as it is.**

**Examples of rounding decimals to the hundredths place:**

3.486 rounds to 3.49

0.0231 rounds to 0.02

12.9057 rounds to 12.91

72.302 rounds to 72.3

**Repeating Decimals**

Sometimes you'll encounter a problem that results in a repeating decimal, decimal numbers that continue to repeat in a certain pattern, no matter how many zeros you add to the dividend. Here is an example:

**Example:**  $6\overline{)0.2}$

You begin to solve the problem as usual, but when you quickly realize the problem will continue to result in a remainder of 2 no matter how many zeros you bring down. This will result in a repeat of the number 3 in the quotient.

If you see that this is what the remainder will continue to be, you can either round off the answer or you can place a bar above two repeating digits to indicate that these digits will continue to repeat indefinitely (drop any extra repeating digits after the two barred digits). Either solution—rounding off or using a bar to indicate repeating decimals—is acceptable.

$$\begin{array}{r} \overline{0.033} \\ 6 \overline{)0.200} \\ \underline{0} \phantom{00} \\ 02 \phantom{00} \\ \underline{0} \phantom{00} \\ 20 \phantom{00} \\ \underline{18} \phantom{00} \\ 20 \phantom{00} \\ \underline{18} \phantom{00} \\ 2 \phantom{00} \end{array}$$

Lesson 6

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## Lesson 6 Dividing Decimals by Decimals

(continued)

When dividing using two decimal fractions, the division process is the same as usual, but the placement of the decimal point is different. Look at the following example:

**Example:**  $1.2 \overline{)20.46}$

**Step 1:** Move the decimal point in the divisor to the right until it is at the end of the number. That means instead of the divisor being 1.2, it's now 12. Since the divisor is now a whole number, it no longer needs a decimal point.

**Step 2:** Next, move the decimal point in the dividend the same number of spaces to the right that you moved it in the divisor. Since you moved it one place to the right in the divisor, you'll move it one place to the right in the dividend. That means the dividend will change from 20.46 to 204.6.

$$12 \overline{)204.6}$$

**Step 3:** Divide as usual, and then bring the decimal directly up into the quotient.

$$\begin{array}{r} 17.05 \\ 12 \overline{)204.60} \\ \underline{12} \phantom{0} \\ 84 \phantom{0} \\ \underline{84} \phantom{0} \\ 06 \phantom{0} \\ \underline{0} \phantom{0} \\ 60 \\ \underline{60} \\ 0 \end{array}$$

Because we've moved the decimal point the same number of places in both the divisor and the dividend, we haven't changed the relationship between the two numbers. When we move the decimal point one place to the right in any number, we're actually multiplying the number by 10. Since

we're multiplying both the divisor and the dividend by the same amount, then the relative value of the two numbers remains the same. Always remember to move the decimal points in both the divisor and the dividend by the same number of places.

Sometimes we'll see a problem where both the divisor and the dividend are less than one. You might see a zero in front of the decimal but usually decimals less than one are written without the "leading" zeros in front; either way, the value is the same.

**Example:**  $.03 \overline{) .46}$

**Step 1:** Turn the divisor into a whole number by moving the decimal in the divisor to the right until it's at the end of the number (.03 becomes 3). Next, move the decimal point the same number of places in the dividend (.46 becomes 46).

$$3 \overline{) 46}$$

**Step 2:** Divide as usual, and bring the decimal point up into the quotient. In this problem, you'll quickly discover that there is a repeating decimal, so you can either round the answer to 15.33 or place a bar over the .33 to show it is a repeating decimal.

$$\begin{array}{r} 15.\overline{33} \\ 3 \overline{) 46.00} \\ \underline{3} \phantom{00} \\ 16 \phantom{00} \\ \underline{15} \phantom{00} \\ 10 \phantom{00} \\ \underline{9} \phantom{00} \\ 10 \phantom{00} \\ \underline{9} \phantom{00} \\ 0 \phantom{00} \end{array}$$

## Lesson 6

(continued)

## Lesson 6 Dividing Whole Numbers by Decimals

(continued)

Finally, let's look at dividing whole numbers by decimals. Keep in mind that everything to the left of a decimal point is a whole number, and everything to the right of a decimal is a fraction. Whole numbers don't need a decimal point, but we can add one and place as many zeros after it as we like without changing the value of the whole number. All of these numbers have the same value:

$$32 \quad 32.0 \quad 32.00 \quad 32.000$$

When we divide a whole number by a fraction, simply add a decimal point and zeros in order to help us solve the problem:

**Example:**  $.4 \overline{)29}$

**Step 1:** Change the whole number in the dividend into a decimal fraction by adding a decimal and zeros. You may want to add more zeros later, but start with two zeros.

$$.4 \overline{)29.0}$$

**Step 2:** Continue as you would to divide a decimal into a decimal. Move the decimal in the divisor and the dividend the same number of spaces:

$$4. \overline{)290.}$$

**Step 3:** Divide as usual. Remember that when you are dividing with decimals, you continue to add zeros until the problem comes out evenly with no remainders. Finally, place the decimal point in the answer, directly above where it now is in the dividend.

$$\begin{array}{r} 72.5 \\ 4. \overline{)290.0} \\ \underline{28} \phantom{0} \\ 10 \phantom{0} \\ \underline{8} \phantom{0} \\ 20 \phantom{0} \\ \underline{20} \\ 0 \end{array}$$





## Lesson 6

(continued)

Does 9 go into 60? No.

Does 10 go into 60? Yes, but we've already written that down so we know we're done.

The factors of 60 are 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, and 60.

### Prime Numbers

While some numbers have several factors, others have just two factors: 1 and the number itself. These numbers are called *prime numbers*. A prime number cannot be divided evenly by any number except itself and 1.

**Example:** What are the factors of 13?

Applying the same technique we used above, see how many numbers will divide evenly into 13.

Does 1 go into 13? Yes, so 1 and 13 are factors.

Does 2 go into 13? No.

Does 3 go into 13? No.

Does 4 go into 13? No.

Does 5 go into 13. No.

Does 6 go into 13? No.

Does 7 go into 13? No.

Once you get past the halfway point, you don't have to go any further because any factors would have already been discovered—we already know 2 doesn't divide evenly into the number.

Since 13 has exactly two factors (1 and 13), it is a prime number.

### New Skills Practice

Complete the following worksheets in your math workbook:

- Lesson 6 New Skills Practice: Dividing Decimals; Factors and Prime Numbers
- Lesson 6 Test

Show all your work and check your answers, reworking any incorrect problems.

**For Enrolled Families**

At the end of this lesson, submit the following three items to your Oak Meadow teacher:

- Lesson 5 Test
- Lesson 6 Test
- Lesson 6 B-test

Make sure the two lesson tests have been graded (by you) and then corrected (by your child). Do not include any of the practice worksheets with your submission.

**Lesson 6***(continued)*


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# Grade 7 Percentages, Simple and Compound Interest



## Mental Math

**Version 1:** You'll need a pair of dice for this game. Roll the dice and form a number. For instance, if you roll a 2 and a 3, you can form either 23 or 32. Think up all the factors of that number. Take note of any numbers that are prime. Do this several times.

**Version 2:** You'll need three dice for this game. Roll two of the dice and form a number with the larger number first. Then roll the third die and divide the original number by the number on the third die. If you have a remainder, express it in the form of a fraction (by putting the remainder over the divisor). For instance, if you first roll a 6 and a 2, you form the number 62. Then if you roll the third die and get a 3, you use it to divide:  $62 \div 3 = 20\frac{2}{3}$ . Next, reverse the digits on your original roll and divide again:  $26 \div 3 = 8\frac{2}{3}$ . Do this several times.

## Skills Check

Complete the following worksheet to practice some of the skills you have learned.

- Lesson 7 Skills Check

## New Skills

### Multiplying and Dividing Decimals by 10, 100, and 1,000

When you multiply or divide any number by 10, 100, or 1,000 (or any whole number formed by 1 with zeros after it), you simply move the decimal point to the left when dividing (which creates a smaller number) or to the right when multiplying (which creates a larger number). The number of spaces we move the decimal point is determined by the number of zeros in the multiplier or the divisor. Let's look at how this works with multiplying first.

## ASSIGNMENT SUMMARY

- Play mental math games.
- Complete the Skills Check worksheet.
- Read New Skills instruction.
- Complete New Skills Practice.
- Complete Lesson 7 Test and Learning Checklist.

## Lesson 7

*(continued)***Example:**  $12 \times 10$ 

There is one zero in the multiplier (10). Since 12 can also be expressed as 12.0, we move the decimal point one place to the right.

$$12 \times 10 = 120$$

**Example:**  $1,532 \times 1,000$ 

There are three zeros in the multiplier (1,000). Since 1,532 can also be expressed as 1,532.000, we move the decimal point three places to the right.

$$1,532 \times 1,000 = 1,532,000$$

This process works the same way when multiplying decimals.

**Example:**  $12.25 \times 10$ 

There is one zero in the multiplier (10), so we move the decimal point one place to the right.

$$12.25 \times 10 = 122.5$$

**Example:**  $32.75 \times 100$ 

There are two zeros in the multiplier (100), so we move the decimal point two places to the right.

$$32.75 \times 100 = 3,275$$

**Example:**  $.31 \times 10,000$ 

There are four zeros in the multiplier, so we move the decimal point four places to the right.

$$.31 \times 10,000 = 3,100$$

Now let's apply this same process to division. Since dividing results in smaller numbers, we need to move the decimal point to the left instead of the right.

**Example:**  $75 \div 10$ 

Remember, all whole numbers can be expressed with a decimal point to the right of the digits—we don't normally place it there unless there is a decimal fraction, but it is assumed to be there at all times. Since there is one zero in the divisor (10), we move the decimal point one place to the left.

$$75 \div 10 = 7.5$$

**Example:**  $23 \div 100$ 

Since there are two zeros in the divisor (100), we move the decimal point two places to the left.

$$23 \div 100 = .23$$

**Example:**  $87 \div 1,000$ 

Since there are three zeros in the divisor (1,000), we move the decimal point three places to the left. Sometimes this means adding zeros as place holders.

$$87 \div 1,000 = .087$$

**Example:**  $5 \div 10,000$ 

Since there are four zeros in the divisor, we move the decimal point four places to the left, adding zeros as necessary.

$$5 \div 10,000 = .0005$$

**Calculating Percentages**

Decimals are one way to express a fraction. A *percentage* is another way of expressing a fraction. The word *percent* means “of a hundred,” meaning the whole is divided into 100 parts.

$$20\% = 20 \text{ parts of } 100 \text{ or } \frac{20}{100}$$

$$85\% = 85 \text{ parts of } 100 \text{ or } \frac{85}{100}$$

To convert a percentage to a decimal, we move the decimal point two places to the left, dividing it by 100. This works because a percentage is based upon 100.

## Lesson 7

*(continued)*

## Lesson 7

*(continued)***Example:** Write 25% as a decimal.

First, remove the percent sign and then move the decimal point two places to the left.

$$25\% = .25$$

**Example:** Write 9% as a decimal.

Remove the percent sign and move the decimal point two places to the left, adding a zero if necessary.

$$9\% = .09$$

The zero acts as a placeholder, showing us the value of the decimal is  $\frac{9}{100}$ . If we didn't add the zero, but instead wrote .9, we would be showing  $\frac{9}{10}$ , not  $\frac{9}{100}$ .

Sometimes we need to calculate the percentage of a certain number. To do this, we change the percent to a decimal and then multiply, as seen in the following examples.

**Example:** How much is 50% of 45?

**Step 1:** Change 50% into a decimal by moving the decimal point two places to the left:  $50\% = .5$

**Step 2:** Multiply the number by the decimal. Put the number with the most digits on top—that just makes it easier.

$$\begin{array}{r} 45 \\ \times .5 \\ \hline 22.5 \end{array}$$

This shows us that 50% of 45 is 22.5.

**Example:** What is 75% of 400?

**Step 1:** Convert the percentage into a decimal:  $75\% = .75$ .

**Step 2:** Multiply the number by the decimal.

$$\begin{array}{r} 400 \\ \times .75 \\ \hline 2000 \\ \hline 2800 \\ \hline 300 \end{array}$$

This shows us that 75% of 400 is 300.

**Example:** What is 3% of 120?

**Step 1:** Convert the percentage into a decimal:  $3\% = .03$ . Remember to be careful about adding the leading zero when converting a single digit percentage to a decimal.

**Step 2:** Multiply the number by the decimal.

$$\begin{array}{r} 120 \\ \times .03 \\ \hline 3.60 \end{array}$$

This shows us that 3% of 120 is 3.6.

We'll often see percentage questions as part of a word problem, like the following example.

**Example:** You-Pick Apple Orchards sold 175 bushels of apples in September, with 95% of the sales going to families who picked apples with their children. How many bushels were picked by families with children in September?

This problem is really asking “What is 95% of 175?” So we follow the steps to find out.

**Step 1:** Convert the percentage into a decimal:  $95\% = .95$ .

**Step 2:** Multiply the number by the decimal.

$$\begin{array}{r} 175 \\ \times .95 \\ \hline 875 \\ \underline{1575} \\ 166.25 \end{array}$$

There were 166.25 bushels of apples picked by families with children in September.

Percentages aren't always expressed as whole numbers, but that doesn't change the process of how we deal with them.

## Lesson 7

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## Lesson 7

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**Example:** On average, 62.5% of book sales at Reader’s Haven come from fiction novels. If the store sold 28 books yesterday, how many of those books were likely to be fiction novels?

This question is really asking, “What is 62.5% of 28?”

**Step 1:** Convert 62.5% into a decimal by moving the decimal point two places to the left: .625.

**Step 2:** Multiply as usual. Since the decimal has more digits, we put it on top.

$$\begin{array}{r} .625 \\ \times 28 \\ \hline 5000 \\ 1250 \\ \hline 175 \end{array}$$

Of all the books sold yesterday, 17.5 were likely to be fiction novels.

(Are you wondering how a store can sell 17.5 books? How can someone sell half a book? The answer is they can’t, of course, but we’re looking at an average amount over time. This means that the store probably sold 17 or 18 fiction books in one day, based on the average of 62.5% of the total book sales. Mathematically, however, the answer comes out to 17.5.)

**Example:** Jessie spent \$32.54 on new shoes and a sales tax of 7.5% was added to this amount. How much was the sales tax? How much was the total bill?

This question is asking, “How much is 7.5% of \$32.54?”

**Step 1:** Convert the percentage into a decimal: .075.

**Step 2:** Multiply as usual. This time you are multiplying two decimals.

$$\begin{array}{r} \$32.54 \\ \times .075 \\ \hline 16270 \\ 22778 \\ \hline \$2.4405 \end{array}$$



**Step 3:** Since we are working with money, we have to round the answer to two places, so \$2.4405 becomes \$2.44. The sales tax came to \$2.44.

**Step 4:** Add the sales tax to the cost of the shoes to get the total:

$$\begin{array}{r} \$32.54 \\ + \quad 2.44 \\ \hline \$34.98 \end{array}$$

The total bill came to \$34.98.

### Interest and Principle

When money is borrowed, usually *interest* is charged; this is basically a fee for using the money. Whenever people borrow money to buy a car, a home, or make a purchase using a credit card, they are obligated to pay back not only the amount they borrowed, which is called the *principal*, but also the interest, which is calculated as a percentage of the principal.

There are two types of interest: simple and compound. We'll introduce *simple interest* first.

Simple interest is an amount calculated at a specific percentage rate for a specific period of time. For example, suppose someone borrows \$10,000 for one year at an interest rate of 12% per year. At the end of the year, the principal (\$10,000) must be repaid along with the accumulated interest (this is often called *accrued interest*). Since 12% of \$10,000 is \$1,200, at the end of the year, the person must pay \$11,200 (\$10,000 principal + \$1,200 interest).

Unless specified otherwise, interest is based upon one year. So if an interest rate is listed as 8%, it means 8% per year, even if the loan period is longer than that. Look at the following example:

**Example:** What is the simple interest on \$20,000 at 8% for three years?

First we calculate the interest for one year:

$$\$20,000 \times 8\% = \$20,000 \times .08 = \$1,600$$

Then we multiply that by three years:

$$\$1,600 \times 3 \text{ years} = \$4,800$$

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The simple interest comes to \$4,800. This is the amount of money that will be paid in addition to repaying the original loan.

**Example:** Jonah loaned Michael \$15,000 for two years at 10% simple interest. How much will Michael owe Jonah at the end of two years?

At the end of two years, Michael will owe Jonah the principal plus the interest. We need to calculate the interest for two years and add that to the principal to determine the total amount due.

**Step 1:** Calculate the interest for one year:

$$\$15,000 \times 10\% = \$15,000 \times .10 = \$1,500$$

**Step 2:** Multiply that by 2 years to get the total interest due.

$$\$1,500 \times 2 \text{ years} = \$3,000$$

**Step 3:** Add the principal and the interest to get the total amount due.

$$\$15,000 + \$3,000 = \$18,000$$

At the end of 2 years, Michael will owe Jonah \$18,000.

### Compound Interest

Simple interest is calculated only on the original principal. Simple interest helps us to understand the concept of interest and principal, but it is rarely used except in small loans between individuals. Businesses, banks, and other financial institutions commonly use *compound interest*, which is calculated on the original principal plus whatever interest has been added each day, month, or year. When we borrow money, the compound interest is added to the amount we have to repay.

Banks usually compound interest on a daily basis, but to simplify the calculations in the examples that follow, we will compound the interest once per year (annually).

**Example:** If you borrowed \$10,000 at 10% interest compounded annually for 3 years, how much would you owe at the end of three years?

**Step 1:** Calculate the interest for the first year.

$$\text{Year 1: } \$10,000 \times 10\% = \$1,000$$

This is the interest for one year. Because it is compounded, we add this to the principal, and this amount becomes the *new principal* for the next year.

$$\$10,000 + 1,000 = \$11,000$$

This is the new principal for Year 2.

**Step 2:** Calculate the interest for the second year, based on the new principal.

$$\text{Year 2: } \$11,000 \times 10\% = \$1,100$$

Once again, we add this interest to the principal.

$$\$11,000 + 1,100 = \$12,100$$

This is the principal for Year 3.

**Step 3:** Repeat the process for the third year.

$$\text{Year 3: } \$12,100 \times 10\% = \$1,210$$

We add this to the principal.

$$\$12,100 + 1,210 = \$13,310$$

The value of \$10,000 compounded annually at 10% for three years is \$13,310.

The simple interest on this amount would have been \$1,000 per year for three years, or \$3,000. The compound interest, which increased each year, came to \$3,310. You can see that compound interest adds up more quickly than simple interest.

Compound interest isn't only used when borrowing money—it can also be earned. Many people put money aside to save, either in a savings account or in another type of account or investment that earns interest. When we put money in an interest-bearing account in a bank, we are actually loaning money to the bank. The bank uses our money to make investments that earn money for the bank. In exchange for the use of our money, the bank agrees to pay us a specified rate of interest, compounded on a regular basis.

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**Lesson 7***(continued)*

**Example:** What is the value of \$50,000 compounded annually at 20% for 5 years?

**Step 1:** Calculate the interest for the first year.

$$\text{Year 1: } \$50,000 \times 20\% = \$10,000$$

Add this interest to the principal; the new principal is \$60,000.

**Step 2:** Calculate the interest for each year, adding the interest to the principal each year to calculate a new principle.

$$\text{Year 2: } \$60,000 \times 20\% = \$12,000$$

Add this interest to the principal; the new principal is \$72,000.

$$\text{Year 3: } \$72,000 \times 20\% = \$14,400$$

Add this interest to the principal; the new principal is \$86,400.

$$\text{Year 4: } \$86,400 \times 20\% = \$17,280$$

Add this interest to the principal; the new principal is \$103,680.

$$\text{Year 5: } \$103,680 \times 20\% = \$20,736$$

Add this interest to the principal; the new principal is \$124,416.

The value of \$50,000 compounded annually at 20% for 5 years would be \$124,416.

**New Skills Practice**

Complete the following worksheets in your math workbook:

- Lesson 7 New Skills Practice: Percentages, Simple and Compound Interest
- Lesson 7 Test

Show all your work and check your answers, reworking any incorrect problems.